NORM INEQUALITIES INVOLVING MATRIX MONOTONE FUNCTIONS

M. Singh and H. L. Vasudeva

Abstract. Let $A, B, X$ be complex matrices with $A, B$ Hermitian positive definite and let $f : (0, \infty) \to (0, \infty)$ be matrix monotone increasing. We prove
\[ (2 + t) \| A^{1/2} (f(A)Xf(B))B \| \leq 2 \| A^2X + tAXB + XB^2 \| \]
and
\[ (2 + t) \| f(A)X + Xf(B) \| \leq 2 f(\lambda) \| A^{1/2}XB^{-1/2} + tA^{1/2}XB^{1/2} + A^{-1/2}XB^{-1/2} \| \]
where $f^{-1}(x) = x(f(x))^{-1}$, $t \in [-2, 2]$ and $\lambda = \min\{\sigma(A), \sigma(B)\}$; $\sigma(A), \sigma(B)$ being the spectrum of $A, B$ respectively and $\| \cdot \|$ any unitarily invariant norm. These inequalities generalize Zhan’s inequalities.


Key words and phrases: Matrix inequalities, Hadamard product, unitarily invariant norms inequalities.

REFERENCES