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NORM INEQUALITIES INVOLVING MATRIX MONOTONE FUNCTIONS

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Abstract. Let A, B, X be complex matrices with A, B Hermitian positive definite and let $f : (0,\infty) \to (0,\infty)$ be matrix monotone increasing. We prove

$$(2+t)\ |||\ A^{\frac{1}{2}}(f(A)Xf^{\perp}(B)+f^{\perp}(A)Xf(B))B^{\frac{1}{2}}\ ||| \leqslant 2\ |||\ A^2X+tAXB+XB^2\ |||$$

and

$$(2+t)\ |||\ f(A)X+Xf(B)\ ||| \leqslant 2\,\frac{f(\lambda)}{\lambda}\ |||\ A^{\frac{3}{2}}XB^{-\frac{1}{2}}+tA^{\frac{1}{2}}XB^{\frac{1}{2}}+A^{-\frac{1}{2}}XB^{\frac{3}{2}}\ |||$$

where $f^{\perp}(x) = x(f(x))^{-1}$, $t \in [-2,2]$ and $\lambda = \min\{\sigma(A), \sigma(B)\}$; $\sigma(A), \sigma(B)$ being the spectrum of A, B respectively and $|||\ .\ |||$ any unitarily invariant norm. These inequalities generalize Zhan's inequalities.

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