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INEQUALITIES BETWEEN $f(\|A\|)$ AND $\|f(|A|)\|$

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Abstract. Let f be a nonnegative concave function on $[0,\infty)$, and let $\|\cdot\|$ be a unitarily invariant norm on the space of $n\times n$ complex matrices. We prove that, for any $n\times n$ complex matrix A, $f(\|A\|)\leqslant\|f(|A|)\|$ provided the norm $\|\cdot\|$ is normalized. On the other hand, if the norm of the identity matrix is 1, then $f(\|A\|)\geqslant\|f(|A|)\|$ for any matrix A. These results extend the theorems of F. Hiai and X. Zhan that were proved in the case when f is an operator monotone function.

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