

MULTIPLE POSITIVE SOLUTIONS OF FOURTH-ORDER BOUNDARY VALUE PROBLEMS

XIAOJIE XU, DAQING JIANG, DONAL O'REGAN AND R. P. AGARWAL

Abstract. In this paper, we discuss the existence of multiple positive solutions for the fourth-order boundary value problem (BVP)

$$u^{(4)}(t) + \beta u''(t) = f(t, u(t)), \quad 0 < t < 1,$$
$$u(0) = u(1) = u''(0) = u''(1) = 0,$$

where $f : [0, 1] \times [0, \infty) \rightarrow [0, \infty)$ is continuous and $\beta < \pi^2$. Existence is established via the theory of fixed point index in cones.

Mathematics subject classification (2000): 34B15.

Key words and phrases: existence; multiple positive solutions; cone; fixed point index.

REFERENCES

- [1] A. R. AFTABIZADEH, *Existence and uniqueness theorems for fourth-order boundary value problems*, J. Math. Anal. Appl. **116** (1986), 415–426.
- [2] R. P. AGARWAL, *On fourth-order boundary value problems arising in beam analysis*, Differential Integral Equations, **26** (1989), 91–110.
- [3] C. P. GUPTA, *Existence and uniqueness results for a bending of an elastic beam equation at resonance*, J. Math. Anal. Appl. **135** (1988), 208–225.
- [4] C. P. GUPTA, *Existence and uniqueness theorem for a bending of an elastic beam equation*, Anal. Appl. **26** (1988), 289–304.
- [5] C. P. GUPTA, *Existence and uniqueness results for some fourth order fully quasilinear boundary value problems*, Anal. Appl. **36** (1990), 169–175.
- [6] M. A. DEL PINO AND R. F. MANASEVICH, *Existence for a fourth-order boundary value problem under a two-parameter nonresonance condition*, Proc. Amer. Math. Soc. **112** (1991), 81–86.
- [7] C. DE COSTER, C. FABRY AND F. MUNYAMARERE, *Nonresonance conditions for fourth order nonlinear boundary value problems*, Internat. J. Math. Sci. **17** (1994), 725–740.
- [8] R. A. USMANI, *A uniqueness theorem for a boundary value problem*, Proc. Amer. Math. Soc. **77** (1979), 327–335.
- [9] Y. YANG, *Fourth-order two-point boundary value problem*, Proc. Amer. Math. Soc. **104** (1988), 175–180.
- [10] D. DUNNINGER, *Existence of positive solutions for fourth-order nonlinear problems*, Boll. Un. Mat. Ital. **7** (1987), 1129–1138.
- [11] P. KORMAN, *A maximum principle for fourth-order ordinary differential equations*, Appl. Anal. **33** (1989), 267–273.
- [12] F. SADYRABAEV, *Two-point boundary value problems for fourth-order*, Acta Univ. Latviensis, **553** (1990), 84–91.
- [13] J. SCHRODER, *Fourth-order two-point boundary value problems; estimates by two side bounds*, Nonlinear Anal. **8** (1984), 107–114.
- [14] R. Y. MA AND H. WANG, *On the existence of positive solutions of fourth-order ordinary differential equations*, Anal. Appl. **59** (1995), 225–231.

- [15] R. Y. MA, J. H. ZHANG AND S. M. FU, *The method of lower and upper solutions for fourth-order two-point boundary value problems*, J. Math. Anal. Appl. **215** (1997), 415–422.
- [16] D. GUO AND V. LAKSHMIKANTHAM, *Nonlinear Problems in Abstract Cones*, Academic Press, New York, 1988.