

## A FUNCTIONAL DIFFERENTIAL EQUATION IN BANACH ALGEBRAS

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*Abstract.* In this paper an existence theorem for the first order functional differential equations in Banach algebras is proved under the mixed generalized Lipschitz and Carathéodory conditions. The existence of extremal solutions is also proved under certain monotonicity conditions.

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