

ON AN ESTIMATE RELATED TO THE HESSIAN AND APPLICATION TO AN OBLIQUE DERIVATIVE PROBLEM

SOFIA GIUFFRÈ

Abstract. We prove an estimate on the $L^2(\Omega)$ -norm of the Hessian of a function $u \in W^{2,q}(\Omega)$, satisfying an oblique derivative type condition on the boundary, allowing the oblique axis to be tangential at a finite number of points of $\partial\Omega$. Using this inequality, the solvability in Sobolev spaces $W^{2,q}(\Omega)$, with q close to 2, follows for a class of nonlinear differential equations in the plane with quadratic growth.

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REFERENCES

- [1] H. AMANN AND M. CRANDALL, *On some existence theorems for semi-linear elliptic equations*, Indiana Univ. Math. J. **27** 5 (1978), 779–790.
- [2] S. CAMPANATO, *On the condition of nearness between operators*, Ann. Mat. Pura Appl. **167** 4 (1994), 243–256.
- [3] S. GIUFFRÈ, *The Nonlinear Oblique Derivative Problem in the Plane*, C.R. Acad. Sci. Paris Sér. I Math. **325** 10 (1997), 1081–1086.
- [4] S. GIUFFRÈ, *Oblique derivative problem for nonlinear elliptic discontinuous operators in the plane with quadratic growth*, C.R. Acad. Sci. Paris Sér. I Math. **328** 10 (1999), 859–864.
- [5] S. GIUFFRÈ, *Well posedness of the tangential oblique derivative problem in the plane*, C.R. Acad. Sci. Paris Sér. I Math. **331** 3 (2000), 207–212.
- [6] S. GIUFFRÈ, *Tangential Oblique Derivative Problem in the Plane with Quadratic Growth*, Comm. Appl. Nonlinear Anal. **10** 2 (2003), 41–54.
- [7] S. GIUFFRÈ, *On an Oblique Derivative Problem of Finite Index for Nonlinear Elliptic Discontinuous equations in the Plane*, Publ. Math. Debrecen **63** 4 (2003), 611–621.
- [8] A. MAUGERI, D. K. PALAGACHEV AND L. SOFTOVA, *Elliptic and parabolic equations with discontinuous coefficients*, Wiley, VCH Publishers, 2000.
- [9] L. NIRENBERG, *On elliptic partial differential equations*, Ann. Scuola Norm. Sup. Pisa **13** 3 (1959), 115–162.
- [10] D. K. PALAGACHEV, *Global strong solvability of Dirichlet problem for a class of nonlinear elliptic equations in the plane*, Le Matematiche **48** 2 (1994), 311–321.
- [11] P. R. POPIVANOV AND D. K. PALAGACHEV, *The degenerate oblique derivative problem for elliptic and parabolic equations*, Akad. Verlag, Berlin, 1997.
- [12] L. SOFTOVA, *An integral estimate for the gradient for a class of nonlinear elliptic equations in the plane*, Z. Anal. Anwendungen **17** 1 (1998), 57–66.
- [13] G. TALENTI, *Problemi di derivata obliqua per equazioni ellittiche in due variabili*, Boll. Un. Mat. Ital. **22** 3 (1967), 505–526.
- [14] N.S. TRUDINGER, *Nonlinear second order elliptic equations*, Lecture Notes of Math. Inst. of Nankai Univ., Tianjin, China, 1986.