

SMOOTHNESS OF ψ -DIRECT SUMS OF BANACH SPACES

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Abstract. Let X_1, X_2, \dots, X_n be Banach spaces and ψ a continuous convex function with some appropriate conditions on a certain convex subset of \mathbb{R}^n . Let $(X_1 \oplus X_2 \oplus \dots \oplus X_n)_\psi$ be the direct sum of X_1, X_2, \dots, X_n equipped with the associated norm with ψ . Then we give the characterization of smoothness of $(X_1 \oplus X_2 \oplus \dots \oplus X_n)_\psi$.

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