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ERGODICITY COEFFICIENT AND PERTURBATION BOUNDS FOR CONTINUOUS–TIME MARKOV CHAINS

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Abstract. For the distribution of a finite, homogeneous, continuous-time Markov chain, we derive perturbation bounds in terms of the ergodicity coefficient of the transition probability matrix. Our perturbation bounds improve upon the known results. We give sensitivity bounds for the coefficient of ergodicity, providing a sufficient condition for the uniqueness of the stationary distribution of the perturbed Markov chain. These results are used to obtain estimates of the speed of convergence for singularly perturbed Markov chains.

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REFERENCES

[1] D. B. ANDREEV, M. A. ELESIN, E. A. KRYLOV, A. V. KUZNETSOV AND A. I. ZEIFMAN, *On ergodicity and stability estimates for some nonhomogeneous Markov chains*, J. Math. Sci. (New York) **112** (2002), 4111–4118.
[2] V. V. ANISIMOV, *Estimates for the deviations of the transition characteristics of nonhomogeneous Markov processes*, Ukraïn. Mat. Zh. **40** (6) (1988), 699–704 (in Russian) (English translation in Ukrainian Math. J. **40** (6) (1988), 588–592).
[3] V. V. ANISIMOV, *Stochastic Processes with a Discrete Component. Limit Theorems*, Vishcha Shkola, Kiev, 1988 (in Russian).
[4] V. V. ANISIMOV AND M. F. TAIROV, *Estimates for the nearness of the transition characteristics of Markov chains*, in: Fourth International Vilnius Conference on Probability Theory and Mathematical Statistics (Abstracts of Communications), Institute of Mathematics and Cybernetics, Vilnius **1** (1985), 28–29 (in Russian).
[5] B. L. GRANOVSKY AND A. I. ZEIFMAN, *Nonstationary Markovian queues*, J. Math. Sci. (New York) **99** (2000), 1415–1438.
[6] R. Z. KHASMINSKII, G. YIN AND Q. ZHANG, *Asymptotic expansions of singularly perturbed systems involving rapidly fluctuating Markov chains*, SIAM J. Appl. Math. **56** (1996), 277–293.
[7] R. Z. KHASMINSKII, G. YIN AND Q. ZHANG, *Constructing asymptotic series for probability distributions with weak and strong interactions*, Quart. Appl. Math. **LV** (1997), 177–200.
[8] A. YU. MITROPHANOV, *Stochastic Markov models for the process of binary complex formation and dissociation*, Mat. Model. **13** (9) (2001), 101–109 (in Russian).
[9] A. YU. MITROPHANOV, *Stability and exponential convergence of continuous-time Markov chains*, J. Appl. Probab. **40** (2003), 970–979.
[10] A. V. RAMESH AND K. S. TRIVEDI, *On the sensitivity of transient solution of Markov models*, in: Proceedings of 1993 ACM SIGMETRICS, Santa Clara (1993), 122–134.
[11] E. SENETA, *Coefficients of ergodicity: structure and applications*, Adv. in Appl. Probab. **11** (1979), 576–590.
[12] E. SENETA, *Explicit forms for ergodicity coefficients and spectrum localization*, Linear Algebra Appl. **60** (1984), 187–197.
[13] G. YIN, Q. ZHANG AND G. BADOWSKI, *Singularly perturbed Markov chains: convergence and aggregation*, J. Multivariate Anal. **72** (2000), 208–229.

[14] Q. ZHANG AND G. YIN, *Markov chains with weak and strong interactions: structural properties*, Nonlinear Anal. **30** (1997), 309–316.
[15] A. I. ZEIFMAN AND D. L. ISAACSON, *On strong ergodicity for nonhomogeneous continuous-time Markov chains*, Stoch. Proc. Appl. **50** (1994), 263–273.
[16] A. I. ZEIFMAN, *Stability of birth-and-death processes*, J. Math. Sci. (New York) **91** (1998), 3023–3031.