

GENERALIZATION OF HILBERT AND HARDY–HILBERT INTEGRAL INEQUALITIES

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Abstract. The main objective of this paper is a study of some new generalizations of Hilbert's and Hardy-Hilbert's type inequalities. We establish general form of multiple Hilbert-type inequality and we also introduce multiple inequality of Hardy-Hilbert type. Further, the best possible constants are obtained for some general cases.

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