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ON WEIGHTED DISCRETE HARDY'S INEQUALITY FOR NEGATIVE POWER NUMBERS

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Abstract. In this paper we consider the weighted discrete Hardy's inequality for different real power numbers $0 \neq r < 1$ and obtain some new refinements of its finite sections. For $r < -1$ our results improve those previously given by Nguyen et al. in [19, 20]. Moreover, we prove that the constant factors involved in the right-hand sides of the obtained inequalities are the best possible, that is, they cannot be replaced with any smaller constant.

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