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GENERAL EULER–BOOLE'S AND DUAL EULER–BOOLE'S FORMULAE

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Abstract. The aim of this paper is to derive general Euler-Boole's and dual Euler-Boole's formulae. More precisely, we derive formulae of Boole type where the integral is approximated not only with the values of the function in certain points but also with values of its derivatives up to $(2n-1)$-th order in end points of the interval. Our method produces formulae of arbitrary degree of exactness. Dual Euler-Boole's formulae are derived by analogy with Simpson's and dual Simpson's rule, and Simpson's 3/8 and Maclaurin's rule. Finally, by analogy with Bullen-Simpson's and Bullen-Simpson's 3/8 inequality, general Bullen-Boole's inequality for a class of $(2k)$-convex functions is derived.

Mathematics subject classification (2000): 26D15, 65D30, 65D32.
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