

## ON THE BEST CONSTANT IN HILBERT'S INEQUALITY

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*Abstract.* The main objective of this paper is a study of some new generalizations of Hilbert's type inequalities. More precisely, we obtain, in some general cases, that the constants involved in the right-hand sides of such inequalities are the best possible.

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