DIFFERENCE DERIVED FROM WEIGHTED HÖLDER’S INEQUALITY

SAICHI IZUMINO, JOSIP PEČARIĆ AND MASARU TOMINAGA

Abstract. We give the maximum of the difference

\[ D_p(a, b; w) := \left( \sum_{k=1}^{n} w_k a_k^p \right)^{\frac{1}{p}} \left( \sum_{k=1}^{n} w_k b_k^q \right)^{\frac{1}{q}} - \sum_{k=1}^{n} w_k a_k b_k \]

derived from a weighted Hörder’s inequality for \( p, q > 1, \ p^{-1} + q^{-1} = 1 \) and for positive \( n \)-tuples \( a := (a_1, \ldots, a_n) \), \( b := (b_1, \ldots, b_n) \) and a weight \( w := (w_1, \ldots, w_n) \) under certain conditions. The discussion in this note is simpler than our previous ones. It comes from the arrangement of a given weight and a linearization of \( D_p(a, b; w) \) via Young’s inequality. As a consequence, we give \( a, b \) and \( w \) which attain the maximum.

Key words and phrases: Hölder’s inequality, convex function, Ozeki’s inequality.

REFERENCES