Mathematical
Inequalities
& Applications
Volume 8, Number 2 (2005), 347–356

ASYMPTOTIC STABILITY AND INTEGRAL INEQUALITIES FOR SOLUTIONS OF LINEAR SYSTEMS ON RADON–NIKODÝM SPACES

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Abstract. We consider the mild solution $u_f(\cdot,0)$ of a well-posed nonhomogeneous Cauchy problem

$$\begin{cases} \dot{u}(t) &= A(t)u(t)+f(t), \quad t\geqslant 0 \\ u(0) &= 0 \end{cases}$$

on a Radon-Nikodým space X, where $A(\cdot)$ is a linear operator-valued function. Under certain additional conditions we will prove that if the homogeneous system

$$\dot{u}(t)=A(t)u(t), \quad t\geqslant 0$$

is exponentially stable, then for each function f belonging to the Sobolev space $W^0_{p1}(\mathbb{R}_+,X)$, $1\leqslant p<\infty$, the solution $u_f(\cdot,0)$ lies in the same space. The converse statement is more subtle, but it certainly works in the autonomous case. Integral inequalities of Landau type for the evolution semigroup associated with the system $(A(t))$ on the space $W^0_{p1}(\mathbb{R}_+,X)$ are also derived.

Mathematics subject classification (2000): 47A10, 47A30, 46N20, 35B35, 26D10.
Key words and phrases: Evolution family, exponential stability, differential inequalities.

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