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REVERSE INEQUALITY TO ARAKI'S INEQUALITY COMPARISON OF $A^pZ^pA^p$ AND $(AZA)^p$

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Abstract. Let A and Z be n-by-n matrices. Suppose $A \geqslant 0$ (positive semi-definite) and $Z > 0$ with extremal eigenvalues a and b. Then, for each $p > 1$, there exist unitary matrices U and V such that

$$\frac{1}{K(a,b,p)}U(AZA)^pU^* \leqslant A^pZ^pA^p \leqslant K(a,b,p)V(AZA)^pV^*.$$

where $K(a,b,p)$ is the Ky Fan constant. The right inequality is both a generalization of Ky Fan's inequality

$$\langle h, Z^p h\rangle \leqslant K(a,b,p)\langle h, Zh\rangle^p,$$

where h is an arbitrary norm one vector, and a reverse inequality to Araki's inequality

$$\|(AZA)^p\| \leqslant \|A^pZ^pA^p\|.$$

for unitarily invariant norms $\|\cdot\|$.

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