REVERSE INEQUALITY TO ARAKI’S INEQUALITY

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Abstract. Let $A$ and $Z$ be $n$-by-$n$ matrices. Suppose $A \succeq 0$ (positive semi-definite) and $Z > 0$ with extremal eigenvalues $a$ and $b$. Then, for each $p > 1$, there exist unitary matrices $U$ and $V$ such that

$$\frac{1}{K(a,b,p)} U(AZA)^p U^* \leq A^pZ^pA^p \leq K(a,b,p) V(AZA)^p V^*.$$

where $K(a,b,p)$ is the Ky Fan constant. The right inequality is both a generalization of Ky Fan’s inequality

$$\langle h, Z^p h \rangle \leq K(a,b,p) \langle h, Z h \rangle^p,$$

where $h$ is an arbitrary norm one vector, and a reverse inequality to Araki’s inequality

$$\| (AZA)^p \| \leq \| A^pZ^pA^p \|.$$

for unitarily invariant norms $\| \cdot \|$.


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REFERENCES