

ON APPROXIMATE t -CONVEXITY

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Abstract. A real valued function f defined on an open convex set D is called $(\varepsilon, \delta, p, t)$ -convex if it satisfies

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) + \delta + \varepsilon|x-y|^p \quad \text{for } x, y \in D.$$

The main result of the paper states that if f is locally bounded from above at a point of D and $(\varepsilon, \delta, p, t)$ -convex (where $0 \leq p < 1$ and $t \leq 1/2$) then it satisfies the convexity-type inequality

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y) + \delta/t + \varepsilon\varphi(\lambda)|x-y|^p \quad \text{for } x, y \in D, \lambda \in [0, 1],$$

where $\varphi : [0, 1] \rightarrow \mathbb{R}$ is a continuous function satisfying

$$\varphi(\lambda) = \max \left\{ \frac{1}{t^p - t}; \frac{1}{(1/2 - t/2)^p - (1-t)^{1-p}(1/2 - t)^p} \right\} (\lambda(1-\lambda))^p.$$

In the case $p = 1, t = 1/2$ analogous results were obtained in [2].

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