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WEIGHTED INEQUALITIES FOR POSITIVE OPERATORS

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Abstract. A technique arising from Schur's Lemma and its converse is shown to generate weighted Lebesgue norm inequalities for a wide class of linear and non-linear positive operators. In many cases the best constants for these inequalities are determined as well. A sharp converse to Schur's Lemma is proved via a minimax principle for a class of positive operators on Banach Function Spaces. This shows that all such inequalities can be generated by this technique and establishes a structure theorem for weight pairs.

Examples involving Hardy and Stieltjes operators are given as well as several Opial-type inequalities. As an illustration of the structure theorem a new proof is given of necessity in the well-known weight characterization for the Hardy operator.

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