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KANTOROVICH TYPE REVERSE INEQUALITIES FOR OPERATOR NORM

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Abstract. In this paper, we shall extend Bourin's theorem for unitarily invariant norm in the framework of operator theory on a Hilbert space by applying the Mond-Pečarić method for convex functions. Moreover we obtain the operator norm version. Among others, we show that if A and Z are positive operators on a Hilbert space H such that $0 < mI \leqslant Z \leqslant MI$ for some scalars $0 < m < M$, then for each $\alpha > 0$

$$\|(AZ^pA)^{\frac{1}{p}}\| \leqslant \alpha\, r(ZA^{\frac{2}{p}}) + \beta(m,M,p,\alpha)\|A\|^{\frac{2}{p}} \qquad \text{for all } p > 1$$

for some suitable constant $\beta(m,M,p,\alpha)$, where $\|\cdot\|$ is the operator norm and $r(\cdot)$ is the spectral radius.

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