

GEOMETRIC MEANS AND HADAMARD PRODUCTS

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Abstract. Ando [1] proved that for m commuting positive definite matrices, the m -fold Hadamard product of their geometric mean is bounded above by their Hadamard product. We obtain a natural extension to the non-commutative case.

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Key words and phrases: arithmetic mean, geometric mean, α -power mean, Hadamard product, tensor product.

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