

## GEOMETRIC MEANS AND HADAMARD PRODUCTS

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*Abstract.* Ando [1] proved that for  $m$  commuting positive definite matrices, the  $m$ -fold Hadamard product of their geometric mean is bounded above by their Hadamard product. We obtain a natural extension to the non-commutative case.

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*Key words and phrases:* arithmetic mean, geometric mean,  $\alpha$ -power mean, Hadamard product, tensor product.

### REFERENCES

- [1] T. ANDO, *Concavity of certain maps on positive definite matrices and applications to Hadamard products*, Linear Algebra Appl., **26**, (1979), 203–241, MR **80f**: 15023.
- [2] R. BHATIA, *Matrix analysis*, Springer–Verlag New York, 1997, MR **98i**: 15003.
- [3] B. Q. FENG, A. M. TONGE, *Matrix versions of some refinements of the arithmetic-geometric mean inequality*, submitted.
- [4] R. A. HORN, C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press 1990, MR **91i**: 15027.
- [5] W. PUSZ, S. L. WORONOWICZ, *Functional calculus for sesquilinear forms and the purification map*, Reports on Mathematical Physics, **8**, (1975), 159–170, MR **54**: # 8316.
- [6] M. SAGAE, K. TANABE, *Upper and lower bounds for the arithmetic-geometric-harmonic means of positive definite matrices*, Linear and Multilinear Algebra, **37**, (1994), 279–282, MR **95m**: 15030.