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A GENERALIZATION OF MACLAURIN'S INEQUALITIES AND ITS APPLICATIONS

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Abstract. The well-known Maclaurin's inequalities are generalized as follows: If x and y are two positive n-tuples, and y and x/y are similarly ordered, then

$$P_n^{[1]}(x)/P_n^{[1]}(y) \geqslant P_n^{[2]}(x)/P_n^{[2]}(y) \geqslant \cdots \geqslant P_n^{[k]}(x)/P_n^{[k]}(y) \geqslant \cdots \geqslant P_n^{[n]}(x)/P_n^{[n]}(y),$$

where $P_n^{[k]}(a)$ is the k-th symmetric mean of a (see[15], p. 283]). The method used in this paper is based on the computational method of descending dimension. As applications, several generalizations for the results of Izumi et al [20], Marshall and Olkin [7], Vasić et al [21], Beesack et al [22], Yang et al [5] are showed.

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REFERENCES

[1] E. F. BEKENBACH, R. BELLMAN, *Inequalities,* Springer-Verlag, Berlin, 1961.
[2] J. C. KUANG, *Applied Inequalities,* Hunan Education Press, 1993. (in Chinese)
[3] W.-L. WANG, P. F. WANG, *A class of inequalities for the symmetric functions*, Acta Math. Sinica, **27**, (1984), 485–497. (in Chinese)
[4] W.-L. WANG, *Some inequalities involving means and their converses*, J.Math. Anal.Appl. **238**, (1999), 567–579.
[5] L. YANG, J. Z. ZHANG, *A class of geometric inequalities on finite points*, Acta Math.Sinica, **23**, (1980), 740–749. (in Chinese)
[6] L. YANG, J. Z. ZHANG, *A class of geometric inequalities concerning the mass-points system*, J. Chinese Univ. of Tech. Sci., **11**, 2 (1981), 1–8. (in Chinese)
[7] A. W. MARSHALL, I. OLKIN AND F. PROSCHAN, *Monotonicity of ratios of means and other applications of majorization*, In Inequalities, 177–190, edited by O.Shisha, New York, London, 1967.
[8] J. J. WEN, Z. LUO, *The optimal generalization of Bernoulli's inequality and its applications*, J.Chengdu Univ.(Natur.Sci.), **20**, 4 (2001), 1–8. (in Chinese)
[9] J. J. WEN, L.LAI AND Z.LUO, *Separation of power mean by symmetric means*, J. Southwest Normal College, (Natur.Sci.), **26**, 3 (2000), 244–250. (in Chinese)
[10] H. N. SHI, J. J. WEN AND B. J. ZHOU, *On an inequality of power means*, Math. Practise and Theory, **31**, 2 (2001), 227–230. (in Chinese)
[11] Z. LUO, L. LAI AND J. J. WEN, *The computational method of descending dimension of global optimizations involving symmetric functions*, J. Chengdu Univ.(Natur.Sci.), **21**, 1 (2002), 5–11. (in Chinese)
[12] D. S. MITRINOVIĆ, J. E. PEČARIĆ AND A. M. FINK, *Classical and new inequalities in analysis*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1993.
[13] R. Z. ZHANG, J. J. WEN, *The multidimensional generalization for Tutecu's conjecture*, J.Sichuan Univ. (Natur.Sci.), **37**, 6 (2000), 803–809. (in Chinese)
[14] Z. LUO, J. J. WEN AND H. N. SHI, *A class of the geometric inequalities involving k-Brocard point*, J. Sichuan Univ. (Natur.Sci.), **39**, 6 (2002), 971–976. (in Chinese)

[15] P. S. BULLEN, D. S. MITRINOVIĆ AND P. M. VASIĆ, *Means and their inequalities*, D. Reidel Publishing Company, 1998.
[16] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, Cambridge, 2th ed. 1952.
[17] J. E. PEČARIĆ AND D. SVRTAN, *Refinements of the Jensen inequalities based on samples with repetitions*, J. Math. Anal. Appl., **222**, (1998), 365–373.
[18] W.-L. WANG, G.-X. LI AND J. CHEN, *Inequalities involving the ratios of means*, J. Chendu Univ. of Tech. Sci., No. 6, (1998), 83–88,90. (in Chinese)
[19] J. J. WEN, H. N. SHI, *Optimizing sharpening for Maclaurin's inequalities*, J. Chengdu Univ. (Natur.Sci.), **19**, 3(2000), 1–8. (in Chinese)
[20] S. IZUMI, K. KOBAYASHI AND T. TAKAHASHI, *On some inequalities*, Proc.Phys. Math.Soc., **16**, 3 (1939), 345–351.
[21] P. M. VASIĆ, I. Ž. MILOVANOVIĆ, *On the ratio of means*, Univ.Beograd Publ. Elektrotehn. Fak. Mat. Fiz. No.577-598, (1977), 33–37.
[22] P. R. BEESACK, J. E. PEČARIĆ, *Integral inequalities of Čebyšev type*, J. Math. Anal. Appl., **111**, (1985), 643–659.
[23] D. S. MITRINOVIĆ, J. E. PEČARIĆ AND V. VOLENCE, *Recent advances in geometric, inequalities*, Chap. 18, Kluwer Academic Publishers, Dordrecht, 1989.