

NECESSARY CONDITIONS FOR SOLVING INITIAL VALUE PROBLEMS WITH INFIMA OF SUPERFUNCTIONS

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Abstract. Goodman proved that the pointwise infimum of all superfunctions is the minimal absolutely continuous solution of

$$x' = f(t, x), \quad t \in [0, 1], \quad x(0) = 0,$$

in case f is a L^1 -bounded Carathéodory function. How far can Carathéodory conditions be weakened without losing that property? First we establish necessary conditions over f for Goodman's method to be valid, and then we use them as a starting point to deduce sufficient ones. In this way we obtain new existence results and we provide new insights concerning the application of Goodman's method.

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