

DIFFERENTIAL DIFFERENCE INEQUALITIES RELATED TO HYPERBOLIC FUNCTIONAL DIFFERENTIAL SYSTEMS AND APPLICATIONS

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Abstract. Initial boundary value problems for quasilinear hyperbolic systems are transformed by discretization in space variables into systems of ordinary functional differential equations. Sufficient conditions for the convergence of the method of lines are given. An implicit difference method is proposed for the numerical solving of systems thus obtained. This leads to an implicit difference method for the original problem. A comparison technique is used. We give a complete convergence analysis for the methods and we show by an example that the new methods are considerable better than the classical schemes.

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