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THE MODIFIED HYERS–ULAM–RASSIAS STABILITY OF A CUBIC TYPE FUNCTIONAL EQUATION

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Abstract. In this paper, we obtain the solution of the following new cubic type functional equation and investigate the modified Hyers-Ulam-Rassias stability of this equation by using the fixed point alternative:

$$f(x+y+2z)+f(x+y-2z)+f(2x)+f(2y)+7f(x)+7f(-x)$$
$$=2[f(x+y)+2f(x+z)+2f(x-z)+2f(y+z)+2f(y-z)].$$

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