AN INEQUALITY FOR MIXED $L^p$-NORMS

HARI BERCOVICI AND DIRK VAN GUCHT

Abstract. Consider a nonnegative measurable function $f$ defined on $\Omega_1 \times \Omega_2$, where $\Omega_j$ is a probability space with probability measure $\mu_j$. We prove the inequality

$$\left[ \int_{\Omega_1 \times \Omega_2} f \, d\mu_1 \, d\mu_2 \right]^p + \int_{\Omega_1 \times \Omega_2} f^p \, d\mu_1 \, d\mu_2 \geq \int_{\Omega_1} \left[ \int_{\Omega_2} f \, d\mu_2 \right]^p \, d\mu_1 + \int_{\Omega_2} \left[ \int_{\Omega_1} f \, d\mu_1 \right]^p \, d\mu_2$$

provided that $1 \leq p \leq 2$. The inequality fails in general if $p > 2$. It also fails if one of the measures $\mu_j$ has total mass greater than one. Curiously however, the inequality is true for all $p \in [1, \infty)$ if the measures $\mu_j$ are counting measures. This last fact follows from a subadditivity result proved by G. A. Raggio for $p$-entropies. Our inequality also has a formulation in terms of $p$-entropies.

Key words and phrases: entropy, partition, refinement, probability measure.

REFERENCES