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SYMMETRIC NORMS AND REVERSE INEQUALITIES TO DAVIS AND HANSEN–PEDERSEN CHARACTERIZATIONS OF OPERATOR CONVEXITY

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Abstract. Let A, B, Z be n-by-n matrices. Suppose $AB \geqslant 0$ (positive semi-definite) and $Z > 0$ with extremal eigenvalues a and b. Then, the sharp inequality

$$\|ZAB\| \leqslant \frac{a+b}{2\sqrt{ab}} \|BZA\|$$

holds for every unitarily invariant norm. Among the consequences, we get the operator inequality $XZX \leqslant [(a+b)^2/4ab]Z$ for every $0 \leqslant X \leqslant I$, and some Kantorovich type inequalities (Mond-Pečarić inequalities). Also in connection, reverse inequalities of Davis and Hansen-Pedersen characterizations of operator convexity are established. For instance, given any operator convex function $f : [0,\infty) \longrightarrow [0,\infty)$ and any subspace $\mathscr{E}$,

$$f(Z_{\mathscr{E}}) \geqslant \frac{4ab}{(a+b)^2} (f(Z))_{\mathscr{E}}.$$

In passing, we point out a simplified proof of Hansen-Pedersen's inequality.

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