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CHARACTERIZATIONS OF CONVEXITY VIA HADAMARD'S INEQUALITY

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Abstract. The classical Hermite–Hadamard inequality, under some weak regularity conditions, characterizes convexity. The aim of the present paper is to give analogous result for the case of generalized convexity induced by two dimensional Chebyshev systems. The basic tool of the proofs is a characterization theorem of continuous, non-convex functions.

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