

SOME OPERATIONS PRESERVING LOG-CONCAVITY OF NONNEGATIVE FUNCTIONS

VADIM E. LEVIT AND EUGEN MANDRESCU

Abstract. In this paper we introduce the notions of k -log-concave and $k, [p, q]$ -concave functions, as generalizations of log-concave and concave sequences, respectively. We also present sufficient conditions under which some operations with such functions yield k -log-concave functions. In particular, we infer that the convolution of a positive 1-log-concave sequence and a nonnegative sequence of $k + 1$ elements is k -log-concave.

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