

GRONWALL INEQUALITIES ON TIME SCALES

FU-HSIANG WONG, CHEH-CHIH YEH AND CHEN-HUANG HONG

Abstract. The authors use several different methods to extend Gronwall's inequality to more general cases on a time scale. Two applications are also given.

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