ON A ČEBYŠEV–TYPE FUNCTIONAL AND GRÜSS–LIKE BOUNDS

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Abstract. The classic Čebyšev functional involves the difference between the integral mean of the product of two functions and the product of the integral means of the individual functions. A Čebyšev-type functional involving the arithmetic average of the upper and lower bounds of one of the functions rather than the integral mean is examined, providing sharp Grüss-like bounds.

The current investigation is undertaken within a measurable space setting. The results are capitalised under a variety of scenarios and in particular in obtaining sharp Grüss-like bounds for perturbed rules in numerical integration.

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