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SHARPENING OF JORDAN'S INEQUALITIES AND ITS APPLICATIONS

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Abstract. In this paper, we establish the following inequalities

$$\frac{\sin r}{r}+\frac{\sin r-r\cos r}{2r^3}(r^2-x^2)\leqslant\frac{\sin x}{x}\leqslant\frac{\sin r}{r}+\frac{r-\sin r}{r^3}(r^2-x^2)$$

for $x\in(0,r], r\leqslant\pi/2$. An application of inequalities above leads to the following refinement of Yang Le inequalities:

$$\begin{aligned}&4C_n^2[\frac{\sin r}{r}\frac{\lambda}{2}\pi+\frac{\sin r-r\cos r}{2r^3}(r^2\frac{\lambda}{2}\pi-\frac{\lambda^3}{8}\pi^3)]^2\cos^2\frac{\lambda}{2}\pi\\ &\leqslant(n-1)\sum_{k=1}^{n}\cos^2\lambda A_k-2\cos\lambda\pi\sum_{1\leqslant i<j\leqslant n}\cos\lambda A_i\cos\lambda A_j\\ &\leqslant 4C_n^2[\frac{\sin r}{r}\frac{\lambda}{2}\pi+\frac{r-\sin r}{r^3}(r^2\frac{\lambda}{2}\pi-\frac{\lambda^3}{8}\pi^3)]^2,\end{aligned}$$

where, $A_i>0(i=1,2,\cdots,n),\sum_{i=1}^{n}A_i\leqslant\pi, 0\leqslant\lambda\leqslant 1$ and $n\geqslant 2$ is a natural number.

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