ON APPROXIMATE DERIVATIONS

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Abstract. Let $\mathscr{A}_1$ be a subalgebra of a Banach algebra $\mathscr{A}$ and let $f : \mathscr{A}_1 \to \mathscr{A}$ satisfies

$$\|f(x+y) - f(x) - f(y)\| \leqslant \delta \quad \text{and} \quad \|f(x \cdot y) - x \cdot f(y) - f(x) \cdot y\| \leqslant \varepsilon,$$

for all $x, y \in \mathscr{A}_1$ and for some constants $\delta, \varepsilon \geqslant 0$. Then we prove that there exists a unique derivation $d : \mathscr{A}_1 \to \mathscr{A}$ such that

$$\|f(x) - d(x)\| \leqslant \delta, \quad x \in \mathscr{A}_1$$

and

$$x \cdot (f(y) - d(y)) = 0, \quad x, y \in \mathscr{A}_1.$$

Moreover, we also prove the Rassias type stability result for derivations.

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