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SCHUR–CONVEXITY FOR A CLASS OF SYMMETRIC FUNCTION AND ITS APPLICATIONS

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Abstract. In this paper, we investigate the symmetric function

$$\prod_n^r(f) = \prod_n^r(f(x)) = \left(\prod_{1\leqslant i_1<i_2<\ldots<i_r\leqslant n} f\left(\frac{1}{r}\sum_{j=1}^{r} x_{i_j}\right)\right)^{\frac{1}{\binom{n}{r}}},$$

where $f(x)$ is a positive function on an interval I. Some analytic inequalities, including "Ky Fan" type inequalities, are established by use of the theory of majorization. An open problem is also solved partly.

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