

AN ALTERNATIVE NOTE ON THE SCHUR-CONVEXITY OF THE EXTENDED MEAN VALUES

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Abstract. The Schur-convex and Schur-concave properties with (x, y) in $(0, \infty) \times (0, \infty)$ for fixed (r, s) of the extended mean values $E(r, s; x, y)$ are researched again, some errors in [F. Qi, J. Sándor, S. S. Dragomir, and A. Sofo, *Notes on the Schur-convexity of the extended mean values*, Taiwanese J. Math. **9**, (3) (2005), 411–420. RGMIA Res. Rep. Coll. **5** (2002), no. 1, Art. 3, 19–27. Available online at URL: <http://rgmia.vu.edu.au/v5n1.html>.] are corrected.

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