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MONOTONICITY OF SEQUENCES INVOLVING CONVEX FUNCTION AND SEQUENCE

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Abstract. Let f be an increasing convex (concave, respectively) function defined on $[0,1]$ and $\{a_i\}_{i\in\mathbb{N}}$ be an increasing positive sequence such that $\left\{i\left(\frac{a_i}{a_{i+1}}-1\right)\right\}_{i\in\mathbb{N}}$ decreases $\left(\left\{i\left(\frac{a_{i+1}}{a_i}-1\right)\right\}_{i\in\mathbb{N}}\right.$ increases, respectively$\left.\right)$, then the sequence $\left\{\frac{1}{n}\sum_{i=1}^{n} f\left(\frac{a_i}{a_n}\right)\right\}_{n\in\mathbb{N}}$ is decreasing.

Let f be an increasing convex (concave, respectively) positive function defined on $[0,1]$ and φ be an increasing convex positive function defined on $[0,\infty)$ such that $\varphi(0)=0$ and the sequence $\left\{\varphi(i)\left[\frac{\varphi(i)}{\varphi(i+1)}-1\right]\right\}_{i\in\mathbb{N}}$ decreases, then the sequence $\left\{\frac{1}{\varphi(n)}\sum_{i=1}^{n} f\left(\frac{\varphi(i)}{\varphi(n)}\right)\right\}_{n\in\mathbb{N}}$ is decreasing.

As applications, taking special sequence $\{a_i\}_{i\in\mathbb{N}}$ and special functions f and φ, many new inequalities between ratios of means are obtained, and the Alzer's inequality, the Minc-Sathre's inequality, and the like, are recovered.

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