Mathematical
Inequalities
& Applications
Volume 9, Number 2 (2006), 273–288

ERROR BOUNDS FOR AFFINE FRACTAL INTERPOLATION

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Abstract. Fractal interpolation constitutes an advance in the techniques of approximation in the sense that the functions used are not necessarily differentiable and show the rough aspect of real-world signals. We prove here that the affine fractal interpolation functions play, for non-smooth functions, a role similar to polynomials for smooth functions. The affine fractal interpolation operator is studied and its linearity and continuity is proven. A sufficient condition for the convergence of this type of interpolant as the step tends to zero is also given. As a consequence, the density of affine fractal functions in the space of continuous functions is deduced. The error of interpolation is bounded in two ways, in terms of the scale factors of the transformation and by means of the Lebesgue constant of the associated partition. Finally, a general method of data fitting is proposed and the validity and convergence of the procedure is proven as well.

Mathematics subject classification (2000): 28A80, 58C05, 65D05, 65D10, 26A18.
Key words and phrases: fractal interpolation functions, iterated function systems.

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Paper MIA-09-28