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STABILITY OF A QUADRATIC FUNCTIONAL EQUATION IN THE SPACE OF DISTRIBUTIONS

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Abstract. We reformulate a quadratic functional equation of the form

$$f(x+y+z)+f(x-y+z)+f(x+y-z)+f(-x+y+z)=4f(x)+4f(y)+4f(z)$$

and an inequality

$$|f(x+y+z)+f(x-y+z)+f(x+y-z)+f(-x+y+z)-4f(x)-4f(y)-4f(z)|\leqslant \varepsilon$$

in the space of distributions. In view of this fact, we use a mollifier and Gauss transform to show that every distributional solution of the inequality is a tempered distribution and finally the stability problem of the equation in the sense of distributions.

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REFERENCES

[1] J. H. BAE, K. W. JUN AND S. M. JUNG, *On the stability of a quadratic functional equation*, Kyungpook Math. J., **43**, (2003), 415–423
[2] P. W. CHOLEWA, *Remarks on the stability of functional equations*, Aequationes Math., **27**, (1984), 76–86
[3] J. CHUNG, *Stability of functional equations in the space of distributions and hyperfunctions*, J. Math. Anal. Appl., **286**, (2003), 177–186
[4] J. CHUNG, *Stability of functional equations in the sense of distributions*, preprint
[5] J. CHUNG, S.-Y. CHUNG AND D. KIM, *The stability of Cauchy equations in the space of Schwartz distributions*, J. Math. Anal. Appl., **295**, (2004), 107–114
[6] S. CZERWIK, *On the stability of quadratic mapping in normed spaces*, Abh. Math. Sem. Univ. Hamburg, **62**, (1992), 59–64.
[7] P. GĂVRUŢĂ, L. CĂDARIU, *The generalized stability of a quadratic functional equation*, preprint
[8] D. H. HYERS, *On the stability of the linear functional equations*, Proc. Nat. Acad. Sci. USA, **27**, (1941), 222–224.
[9] G. H. KIM, *On the stability of the quadratic mapping in normed spaces*, Int. J. Math. Math. Sci., **25**, 4 (2001), 217–299
[10] T. MATSUZAWA, *A calculus approach to hyperfunctions* III, Nagoya Math. J., **118**, (1990), 133–153.
[11] F. SKOF, *Local properties and approximation of operators*, Geometry of Banach spaces and related topics, Rend. Sem. Mat. Fis. Milano, **53**, (1983), 113–129.

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