Mathematical
Inequalities
& Applications
Volume 9, Number 2 (2006), 343–349

ON AN IMPROVEMENT OF BERNSTEIN'S POLYNOMIAL INEQUALITIES

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Abstract. The following Bernstein inequality

$$\max_{|z|\leqslant 1} |p'(z)| \leqslant n \max_{|z|\leqslant 1} |p(z)|,$$

valid for all complex polynomials p of degree n, has been extended by Ruscheweyh to

$$\max_{|z|\leqslant 1} |p'(z)| \leqslant n \max_{|z|\leqslant 1} |p(z)| - \frac{2n}{n+2}|p(0)|, \quad n \geqslant 2.$$

We prove in this note that two other Bernstein inequalities, i.e.,

$$\max_{-1\leqslant x\leqslant 1} \left|\sqrt{1-x^2}p'(x)\right| \leqslant n \max_{-1\leqslant x\leqslant 1} |p(x)|$$

or

$$\max_{0\leqslant\theta\leqslant 2\pi} |t'(\theta)| \leqslant n \max_{0\leqslant\theta\leqslant 2\pi} |t(\theta)|,$$

where $t(\theta)$ is a complex trigonometric polynomial of degree n do not admit similar extensions. In addition we obtain a new proof of Marcel Riesz interpolation formula.

Mathematics subject classification (2000): 41A17.
Key words and phrases: Bernstein inequalities, Chebyshev polynomial, Riesz interpolation formula, bound-preserving functions.

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