

SOME OPTIMIZATION PROBLEMS FOR THE MINIMAL ANNULUS OF A CONVEX SET

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Abstract. In this paper we relate the minimal annulus of a planar convex body K with the six classic geometric measures associated with it. First, we obtain all the possible bounds (upper and lower bounds) for the measures A, p, D, ω, R_K and r_K of a convex body K with given minimal annulus. Then, we solve the problem of maximizing and minimizing the area and the perimeter of convex bodies with given circumradius and minimal annulus. We prove the optimal inequalities for each of those problems, determining also its corresponding extremal sets.

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