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WEIGHTED INEQUALITIES FOR HARMONIC MEANS

P. ORTEGA SALVADOR AND C. RAMÍREZ TORREBLANCA

Abstract. We characterize the weighted weak and strong type (p,q) inequalities for the harmonic averaging operator

$$Tf(x) = \frac{x}{\int_0^x \frac{1}{f}}$$

in the cases $0 < p \leqslant q < \infty$ and $0 < q < p < \infty$.

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