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THE BEST BOUNDS IN GAUTSCHI–KERSHAW INEQUALITIES

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Abstract. By employing the convolution theorem of Laplace transforms, some asymptotic formulas and integral representations of the gamma, psi and polygamma functions, and other analytic techniques, this note provides an alternative proof of a monotonicity and convexity property by N. Elezović, C. Giordano and J. Pečarić in [4] to establish the best bounds in Gautschi-Kershaw inequalities. Moreover, some (logarithmically) complete monotonicity results on functions related to Gautschi-Kershaw inequalities are remarked.

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REFERENCES

[1] M. ABRAMOWITZ, I. A. STEGUN (Eds), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series **55**, 4th printing, with corrections, Washington, 1965.

[2] C. BERG, *Integral representation of some functions related to the gamma function*, Mediterr. J. Math. **1**, (4) (2004), 433–439.

[3] CH.-P. CHEN, F. QI, *The best bounds in Wallis' inequality*, Proc. Amer. Math. Soc. **133**, (2) (2005), 397–401. RGMIA Res. Rep. Coll. **5**, (4) (2002), Art. 13. *URL:* `http://rgmia.vu.edu.au/v5n4.html`.

[4] N. ELEZOVIĆ, C. GIORDANO AND J. PEČARIĆ, *The best bounds in Gautschi's inequality*, Math. Inequal. Appl. **3**, (2) (2000), 239–252.

[5] B.-N. GUO, F. QI, *Inequalities for generalized weighted mean values of convex function*, Math. Inequal. Appl. **4**, (2) (2001), 195–202. RGMIA Res. Rep. Coll. **2**, (7) (1999), Art. 11, 1059–1065. Available online at *URL:* `http://rgmia.vu.edu.au/v2n7.html`.

[6] W. MAGNUS, F. OBERHETTINGER AND R. P. SONI, *Formulas and Theorems for the Special Functions of Mathematical Physics*, Springer, Berlin, 1966.

[7] F. QI, *A note on Schur-convexity of extended mean values*, Rocky Mountain J. Math., **35**, (5) (2005), 1787–1793.

[8] F. QI, *Generalized abstracted mean values*, J. Inequal. Pure Appl. Math., **1**, (1) (2000), Art. 4. Available online at *URL:* `http://jipam.vu.edu.au/article.php?sid=97`. RGMIA Res. Rep. Coll. **2**, (5) (1999), Art. 4, 633–642. Available online at *URL:* `http://rgmia.vu.edu.au/v2n5.html`.

[9] F. QI, *Generalized weighted mean values with two parameters*, R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci. **454**, (1978) (1998), 2723–2732.

[10] F. QI, *Logarithmic convexity of extended mean values*, Proc. Amer. Math. Soc., **130**, (6) (2002), 1787–1796. RGMIA Res. Rep. Coll. **2**, (5) (1999), Art. 5, 643–652. Available online at *URL:* `http://rgmia.vu.edu.au/v2n5.html`.

[11] F. QI, *Monotonicity results and inequalities for the gamma and incomplete gamma functions*, Math. Inequal. Appl. **5**, (1) (2002), 61–67. RGMIA Res. Rep. Coll. **2**, (7) (1999), Art. 7, 1027–1034. Available online at *URL:* `http://rgmia.vu.edu.au/v2n7.html`.

[12] F. QI, *Schur-convexity of the extended mean values*, RGMIA Res. Rep. Coll., **4**, (4) (2001), Art. 4, 529–533. Available online at *URL:* `http://rgmia.vu.edu.au/v4n4.html`.

[13] F. QI, *The extended mean values: definition, properties, monotonicities, comparison, convexities, generalizations, and applications*, Cubo Mat. Educ. **5**, (3) (2003), 63–90. RGMIA Res. Rep. Coll. **5**, (1) (2002), Art. 5, 57–80. Available online at *URL:* `http://rgmia.vu.edu.au/v5n1.html`.

[14] F. QI, CH.-P. CHEN, *A complete monotonicity property of the gamma function*, J. Math. Anal. Appl. **296**, (2) (2004), 603–607.

[15] F. QI, B.-N. GUO, *Complete monotonicities of functions involving the gamma and digamma functions*, RGMIA Res. Rep. Coll., **7**, (1) (2004), Art. 8, 63–72. Available online at *URL:* `http://rgmia.vu.edu.au/v7n1.html`.

[16] F. QI, B.-N. GUO AND CH.-P. CHEN, *Some completely monotonic functions involving the gamma and polygamma functions*, J. Austral. Math. Soc., **80**, (1) (2006), 81–88. RGMIA Res. Rep. Coll. **7**, (1) (2004), Art. 5, 31–36. Available online at *URL:* `http://rgmia.vu.edu.au/v7n1.html`.

[17] F. QI, J. SÁNDOR, S. S. DRAGOMIR AND A. SOFO, *Notes on the Schur-convexity of the extended mean values*, Taiwanese J. Math. **9**, (3) (2005), 411–420. RGMIA Res. Rep. Coll. **5**, (1) (2002), Art. 3, 19–27. Available online at *URL:* `http://rgmia.vu.edu.au/v5n1.html`.

[18] F. QI, S.-L. XU, *The function* $(b^x - a^x)/x$*: Inequalities and properties*, Proc. Amer. Math. Soc. **126**, (11) (1998), 3355–3359.

[19] F. QI, SH.-Q. ZHANG, *Note on monotonicity of generalized weighted mean values*, R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci. **455**, (1989) (1999), 3259–3260.

[20] ZH.-X. WANG, D.-R. GUO, *Special Functions*, Translated from the Chinese by D.-R. Guo and X.-J. Xia, World Scientific Publishing, Singapore, 1989.

[21] ZH.-X. WANG, D.-R. GUO, *Téshū Hánshú Gáilùn* (*A Panorama of Special Functions*), The Series of Advanced Physics of Peking University, Peking University Press, Beijing, China, 2000. (Chinese)

[22] D. V. WIDDER, *The Laplace Transform*, Princeton University Press, Princeton, 1941.