

ON (H_{pq}, L_{pq}) -TYPE INEQUALITY OF MAXIMAL OPERATOR OF MARCINKIEWICZ–FEJÉR MEANS OF DOUBLE FOURIER SERIES WITH RESPECT TO THE KACZMARZ SYSTEM

G. GÁT, U. GOGINAVA AND K. NAGY

Abstract. The main aim of this paper is to prove that the maximal operator of Marcinkiewicz-Fejér means of double Fourier series with respect to the Kaczmaz system is bounded from the dyadic Hardy-Lorentz space H_{pq} into the Lorentz space L_{pq} for every $p > \frac{1}{2}$ and $0 < q \leq \infty$ provided that the supremum in the maximal operator is taken over special indices. As a consequence, we obtain the a.e. convergence of Marcinkiewicz-Fejér means of double Fourier series for special indices with respect to the Walsh-Kaczmaz system. That is, $\sigma_{2^n}(f, x^1, x^2) \rightarrow f(x^1, x^2)$ a.e. as $n \rightarrow \infty$.

Mathematics subject classification (2000): 42C10.

Key words and phrases: Walsh-Kaczmaz system, Marcinkiewicz-Fejér means, maximal operator.

REFERENCES

- [1] M.I. DYACHENKO, On the (C, α) -summability of multiple trigonometric Fourier series, *Soobshch. Akad. Nauk Gruzii* **131**, (1988), 261–263.
- [2] G. GÁT, On $(C, 1)$ summability of integrable functions with respect to the Walsh-Kaczmaz system, *Studia Math.* **130**, (1998), 135–148.
- [3] G. GÁT, Convergence of Marcinkiewicz means of integrable functions with respect to two-dimensional Vilenkin systems, *Georgian Mathematical Journal* **11**, (2004), No. 3, 467–478.
- [4] U. GOGINAVA, Marcinkiewicz-Fejér means of d -dimensional Walsh-Fourier series, *Math. Anal. and Appl.* **307**, (2005), no. 1, 206–218.
- [5] K. NAGY, Some convergence properties of the Walsh-Kaczmaz system with respect to the Marcinkiewicz means, *Rendiconti del Circolo Matematico di Palermo, Serie II, Suppl.* **76**, (2005), 503–516.
- [6] J. MARCINKIEWICZ, Sur une methode remarquable de sommation des series doubleles de Fourier, *Ann. Scuola Norm. Sup. Pisa* **8**, (1939), 149–160.
- [7] F. SCHIPP, W. R. WADE, P. SIMON AND J. PÁL, *Walsh Series, An Introduction to Dyadic Harmonic Analysis*, Adam Hilger (Bristol-New York 1990).
- [8] F. SCHIPP, Pointwise convergence of expansions with respect to certain product systems, *Analysis Math.* **2**, (1976), 63–75.
- [9] P. SIMON, On the Cesaro summability with respect to the Walsh-Kaczmaz system, *J. Approx. Theory.* **106**, (2000), 249–261.
- [10] V. A. SKVORCOV, On Fourier series with respect to the Walsh-Kaczmaz system, *Analysis Math.* **7**, (1981), 141–150.
- [11] A. A. ŠNEIDER, On series with respect to the Walsh functions with monotone coefficients, *Izv. Akad. Nauk SSSR Ser. Math.* **12**, (1948), 179–192.
- [12] W. S. YOUNG, On the a.e. convergence of Walsh-Kaczmaz-Fourier series, *Proc. Amer. Math. Soc.* **44**, (1974), 353–358.
- [13] F. WEISZ, Convergence of double Walsh-Fourier series and Hardy spaces, *Approx. Theory and its Appl.* **17**, (2001), 32–44.

- [14] F. WEISZ, Martingale Hardy spaces and their applications in Fourier analysis, Springer-Verlag, Berlin, 1994.
- [15] L. V. ZHIZHIASHVILI, Generalization of a theorem of Marcinkiewicz, *Izv. Akad. nauk USSR Ser Math.* **32**, (1968), 1112–1122.