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GENERALIZATION OF THE KANTOROVICH TYPE OPERATOR INEQUALITIES VIA GRAND FURUTA INEQUALITY

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Abstract. In this note we show the characterization of the δ-order by means of a generalized Kantorovich constant via Grand Furuta inequality, which is an extension result of that from M. Fujii, E. Kamei and Y. Seo, *Kantorovich type operator inequalities via grand Furuta inequality*, Sci. Math., **3**, (2000), 263–272. Among other, we show the following characterization of the δ-order: Let A, B be positive invertible operators on a Hilbert space H satisfying $MI \geqslant A \geqslant mI > 0$ and $NI \geqslant B \geqslant nI > 0$. Then the following statements are mutually equivalent for each $\delta \in [0,1]$:

(i) $A^\delta \geqslant B^\delta$,

(ii) $K(n^r, N^r, 1+\frac{p-\delta}{r}, 1+\frac{q-\delta}{r})A^q \geqslant B^p$ for all $p > \delta, q > \delta$ and $r > \delta$,

(iii) $\overline{K}(m^r, M^r, 1+\frac{q-\delta}{r}, 1+\frac{p-\delta}{r})A^q \geqslant B^p$ for all $p > \delta, q > \delta$ and $r > \delta$,

where the case $\delta = 0$ means the chaotic order $\log A \geqslant \log B$.

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REFERENCES

[1] J. I. FUJII, Y. SEO, *Characterizations of chaotic order associated with the Mond-Shisha difference*, Math. Inequal. Appl., **5**, (4) (2002), 725–734.
[2] M. FUJII, M. HASHIMOTO, Y. SEO AND M. YANAGIDA, *Characterizations of usual and chaotic order via Furuta and Kantorovich inequalities*, Sci. Math., **3**, (2000), 405–418.
[3] M. FUJII, E. KAMEI AND Y. SEO, *Kantorovich type operator inequalities via grand Furuta inequality*, Sci. Math., **3**, (2000), 263–272.
[4] T. FURUTA, *$A \geqslant B \geqslant 0$ assures $(B^r A^p B^r)^{1/q} \geqslant B^{(p+2r)/q}$, for $r \geqslant 0$, $p \geqslant 0$, $q \geqslant 1$ with $(1+2r)q \geqslant p+2r$*, Proc. Amer. Math. Soc., **101**, (1987), 85–88.
[5] T. FURUTA, *Extension of the Furuta inequality and Ando-Hiai log-majorization*, Linear Algebra Appl., **219**, (1995), 139–155.
[6] T. FURUTA, *Operator inequalities associated with Hölder-McCarthy and Kantorovich inequalities*, J. Inequal. Appl., **2**, (1998), 137–148.
[7] T. FURUTA, M. GIGA, *A complementary result of Kantorovich type order preserving inequalities by Mićić-Pečarić-Seo*, Linear Algebra Appl., **369**, (2003), 27–40.
[8] T. FURUTA, Y. SEO, *An application of generalized Furuta inequality to Kantorovich type inequalities*, Sci. Math., **2**, (1999), 393–399.
[9] M. HASHIMOTO, M. YANAGIDA, *Further extensions of characterizations of chaotic order associated with Kantorovich type inequalities*, Sci. Math., **3**, (2000), 127–136.
[10] J. MIĆIĆ, J. PEČARIĆ AND Y. SEO, *Function order of positive operators based on the Mond-Pečarić method*, Linear Alg. Appl., **360**, (2003), 15–34.

[11] J. MIĆIĆ, J. PEČARIĆ, *Characterizations of δ–order associated with Kantorovich type operator inequalities*, Sci. Math. Japon., **63**, (2006), 149–155.
[12] Y. SEO, *A characterization of operator order via grand Furuta inequality*, J. Inequal. Appl., **6**, (2001), 473–481.
[13] T. YAMAZAKI, M. YANAGIDA, *Characterizations of chaotic order associated with Kantorovich inequality*, Sci. Math., **2**, (1999), 37–50.