

SCHUR-CONVEXITY OF THE COMPLETE SYMMETRIC FUNCTION

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Abstract. This paper investigates Schur-convexity of the complete symmetric function $c_r(x) = \sum_{i_1+\dots+i_n=r} x_1^{i_1} \dots x_n^{i_n}$ and the function $\phi_r(x) = \frac{c_r(x)}{c_{r-1}(x)}$, where i_1, \dots, i_n are non-negative integers and $r \geq 1$. Some inequalities, including Ky Fan type inequality, are established by use of the theory of majorization. It is also concerned with an open problem proposed by Menon [1].

Mathematics subject classification (2000): 05E05, 26E60, 26D20.

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