

HIGHER DIMENSIONAL COMPACTNESS OF HARDY OPERATORS INVOLVING OINAROV–TYPE KERNELS

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Abstract. The compactness of the higher dimensional generalized Hardy operator $(\mathcal{K}f)(x) = \int_{S_x} k(x, y)f(y)dy$ and its conjugate operator \mathcal{K}^* has been characterized for the case $1 < p, q < \infty$. This is done by reducing the problem to the corresponding one dimensional situation.

Mathematics subject classification (2000): 46E35, 26D10.

Key words and phrases: Hardy operator, Oinarov kernel, boundedness, compactness.

REFERENCES

- [1] T. ANDO, *On the compactness of integral operators*, Proc. Ned. Akad. Van Wet., **24**, (1962), 235–239.
- [2] S. BLOOM, R. KERMAN, *Weighted norm inequalities for operators of Hardy type*, Proc. Amer. Math. Soc., **113**, (1991), 135–141.
- [3] B. GUPTA, *Mapping Properties of Certain Integral Averaging Operators*, Doctoral Thesis, 2004.
- [4] P. JAIN, B. GUPTA, *Compactness of Hardy-Steklov operator*, J. Math. Anal. Appl., **288**, (2003), 680–691.
- [5] P. JAIN, P. K. JAIN AND B. GUPTA, *Compactness of Hardy-type operators over star-shaped regions in \mathbb{R}^N* , Canad. Math. Bull., **47**, (4) (2004), 540–552.
- [6] A. KUFNER, L. E. PERSSON, *Weighted Inequalities of Hardy Type*, World Scientific, 2003.
- [7] E. LOMAKINA, V. D. STEPANOV, *On the Hardy-type integral operators in Banach function spaces*, Publ. Math., **42**, (1998), 165–194.
- [8] V. G. MAZ'JA, *Sobolev Spaces*, Springer Verlag, Springer Series in Soviet Mathematics, 1985.
- [9] R. OINAROV, *Weighted inequalities for a class of integral operators* (Russian), Dokl. Akad. Nauk. SSSR, **319**, (1991), 1076–1078; translation in Soviet Math. Dokl., **44**, (1992), 291–293.
- [10] R. OINAROV, *Two-sided estimates for the norm of some classes of integral operators* (Russian), Trudy Mat. Inst. Steklov, **204**, (1993), 240–250; translation in Proc. Steklov Inst. Math., **204**, (1994), 205–214.
- [11] G. SINNAMON, *A weighted gradient inequality*, Proc. Royal Soc. Edinburgh Sect. A, **111**, (1989), 329–335.
- [12] G. SINNAMON, *One dimensional Hardy-inequalities in many dimensions*, Proc. Royal Soc. Edinburgh Sect. A, **128A**, (1998), 833–848.
- [13] A. WEDESTIG, *Weighted Inequalities of Hardy-Type and their Limiting Inequalities*, Doctoral Thesis, 2003.