

BOUNDS FOR THE RATIO AND DIFFERENCE BETWEEN PARALLEL SUM AND SERIES VIA MOND–PEČARIĆ METHOD

J. I. FUJII, M. NAKAMURA, J. PEČARIĆ AND Y. SEO

Abstract. Upper bounds for the ratio and the difference between parallel sum and series of operator connections in the sense of Anderson-Duffin-Trapp are obtained, in which the Mond-Pečarić method for convex functions is applied: Let A and B be positive operators on a Hilbert space such that $0 < mI \leq A, B \leq MI$ for some scalars $m < M$. Then we show an upper bound of the difference of parallel sum and series :

$$(A + B) - (A : B) \leq 2 \left(M + m - \sqrt{Mm} \right) I.$$

As an application, we show a noncommutative Kantorovich inequality: For positive operators A and B such that $0 < mI \leq A, B \leq MI$,

$$\frac{A + B}{2} \leq \frac{(M + m)^2}{4Mm} \left(\frac{A^{-1} + B^{-1}}{2} \right)^{-1}$$

and moreover we show the following refinement:

$$\frac{2\sqrt{Mm}}{M + m} \frac{A + B}{2} \leq A \sharp B \leq \frac{M + m}{2\sqrt{Mm}} \left(\frac{A^{-1} + B^{-1}}{2} \right)^{-1},$$

where $A \sharp B$ is the geometric mean.

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