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DUALITY IN NONLINEAR COMPLEMENTARITY THEORY BY USING INVERSIONS AND SCALAR DERIVATIVES

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Abstract. The notion of infinitesimal exceptional family of elements will be introduced. By using a special inversion mapping a duality between the exceptional family of elements and the infinitesimal exceptional family of elements will be presented. By using this duality and the notion of scalar derivatives existence theorems for complementarity problems in Hilbert spaces will be presented. Remark (important!): The notion of duality will be introduced not for the sake of "playing" with a new notion, but in order to prove Theorem 8.8, which provides a powerful tool for solving complementarity problems.

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