

THE HAMY SYMMETRIC FUNCTION AND ITS GENERALIZATION

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Abstract. In the paper we investigate and generalize the Hamy symmetric function: $F_n(x, r) = \sum_{1 \leq i_1 < i_2 < \dots < i_r \leq n} \left(\prod_{j=1}^r x_{i_j} \right)^{\frac{1}{r}}$. The Schur-convexity is discussed and some analytic inequalities are established by use of the theory of majorization.

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