

A REMARK ON AN INEQUALITY FOR THE PRIME COUNTING FUNCTION

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Abstract. We note that the inequalities $0.92 \frac{x}{\log(x)} < \pi(x) < 1.11 \frac{x}{\log(x)}$ do not hold for all $x \geq 30$, contrary to some references. These estimates on $\pi(x)$ came up recently in papers on algebraic number theory.

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