# FUZZY RANDOM KOROVKIN THEORY AND INEQUALITIES

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*Abstract.* We introduce and study the fuzzy random positive linear operators acting on fuzzy random continuous functions. We establish a series of fuzzy random Shisha–Mond type inequalities of $L^q$-type $1 \leqslant q < \infty$ and related fuzzy random Korovkin type theorems, regarding the fuzzy random $q$-mean convergence of fuzzy random positive linear operators to the fuzzy random unit operator for various cases. All convergences are with rates and are given via the above fuzzy random inequalities involving the fuzzy random modulus of continuity of the engaged fuzzy random function. The assumptions for the Korovkin theorems are minimal and of natural realization, fulfilled by almost all example fuzzy random positive linear operators. The astonishing fact is that the real Korovkin test functions assumptions are enough for the conclusions of our fuzzy random Korovkin theory. We give at the end applications.

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