ON A PROBLEM OF UNIVALENCE OF FUNCTIONS SATISFYING A DIFFERENTIAL INEQUALITY

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Abstract. Let $\mathcal{H}_\alpha(\beta)$ denote the class of normalized functions $f$, analytic in the unit disc $E$, which satisfy the condition

$$\text{Re} \left[ (1 - \alpha)f'(z) + \alpha \left( 1 + \frac{zf''(z)}{f'(z)} \right) \right] > \beta, \ z \in E,$$

where $\alpha$ and $\beta$ are pre-assigned real numbers. H. S. Al-Amiri and M. O. Reade, in 1975, have shown that for $\alpha \leq 0$ and also for $\alpha = 1$, the functions in $\mathcal{H}_\alpha(0)$ are univalent in $E$. In 2005, V. Singh, S. Singh and S. Gupta proved that for $0 < \alpha < 1$, functions in $\mathcal{H}_\alpha(\alpha)$ are also univalent. In the present note, we establish that functions in $\mathcal{H}_\alpha(\beta)$ are univalent for all real numbers $\alpha$ and $\beta$ satisfying $\alpha \leq \beta < 1$ and that the result is sharp in the sense that the constant $\beta$ cannot be replaced by any real number less than $\alpha$.


Key words and phrases: univalent function, convex function, differential subordination.

REFERENCES