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ON A PROBLEM OF UNIVALENCE OF FUNCTIONS SATISFYING A DIFFERENTIAL INEQUALITY

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Abstract. Let $\mathcal{H}_\alpha(\beta)$ denote the class of normalized functions f, analytic in the unit disc E, which satisfy the condition

$$\operatorname{Re}\left[(1-\alpha)f'(z)+\alpha\left(1+\frac{zf''(z)}{f'(z)}\right)\right]>\beta,\ z\in E,$$

where α and β are pre-assigned real numbers. H. S. Al-Amiri and M. O. Reade, in 1975, have shown that for $\alpha \leqslant 0$ and also for $\alpha = 1$, the functions in $\mathcal{H}_\alpha(0)$ are univalent in E. In 2005, V. Singh, S. Singh and S. Gupta proved that for $0<\alpha<1$, functions in $\mathcal{H}_\alpha(\alpha)$ are also univalent. In the present note, we establish that functions in $\mathcal{H}_\alpha(\beta)$ are univalent for all real numbers α and β satisfying $\alpha \leqslant \beta < 1$ and that the result is sharp in the sense that the constant β cannot be replaced by any real number less than α.

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