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ON THE HYERS–ULAM–RASSIAS STABILITY OF A n–DIMENSIONAL QUADRATIC FUNCTIONAL EQUATION

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Abstract. Let $n \geqslant 2$ be an integer. In this paper, we investigate the generalized Hyers-Ulam-Rassias stability of a n-dimensional quadratic functional equation on Banach spaces and Banach modules over a Banach algebra;

$$(4-n)f\left(\sum_{j=1}^{n} x_j\right) + \sum_{i=1}^{n} f\left(\sum_{j=1}^{n} \theta(i,j)x_j\right) = 4\sum_{i=1}^{n} f(x_i),$$

where the function θ is defined by $\theta(i,j) = \begin{cases} 1 & \text{if } i \neq j \\ -1 & \text{if } i = j \end{cases}$.

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